MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

EEL1176 – FIELD THEORY
(All sections / Groups)

22 OCTOBER 2018
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 8 pages with 4 questions only.
2. Answer all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

Question 1 (25 marks)

(a) Given vector $\vec{A} = y\hat{x} + (x+z)\hat{y}$ and point P is located at (-2,6,3). Determine the following:

(i) Express the location of point P in spherical coordinates.

[3 marks]

(ii) Evaluate vector $\vec{A}$ at point P in the spherical system.

[7 marks]

(b) A structure with the following dimensions are provided: $2 \le r \le 6$; $\frac{\pi}{4} \le \varphi \le \pi$; $z = 0$. Determine the total surface area by performing surface integral.

[4 marks]

(c) Given a vector field $\bar{D} = 3R^2\hat{R}$. The flux $\oiint_S \bar{D} \cdot ds = 180\pi$ for the region enclosed between the spherical shells defined by $R = 1$ and $R = 2$ as seen in Figure 1 (c). Answer the following:

(i) State the Divergence Theorem mathematically.

[1 mark]

(ii) Verify the Divergence theorem by determining the volume integral of the divergence $\iiint_v \nabla \cdot \bar{D} dv$ for the given region.

[10 marks]

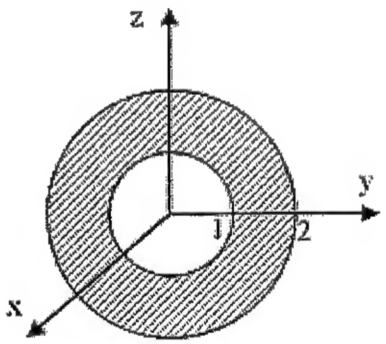

Figure 1(c)

Continued ...

Question 2 (25 marks)

(a) A positively charged rod is brought close to a neutral piece of paper. The charged rod attracts the paper. Draw a diagram showing the separation of charge in the paper, and explain why attraction occurs.

[4 marks]

(b) Two point charges $Q_1 = 3\ \mu C$ and $Q_2 = -4\mu C$ are placed at (3, 2, 1) and (-4, 0, 6), respectively.

(i) Determine the force, $\vec{F}$ on the charge 20μC at the origin (0, 0, 0).

[4 marks]

(ii) Find the electric field intensity, $\vec{E}$ on the charge 20μC at the origin (0, 0, 0).

[2 marks]

(c) A spherical shell of radius a shown in Figure 2(c) contains a uniform charge density of $\rho_v / 2$ (C/m^3). Determine the electric flux density, **D** by the given Gaussian surfaces for each of the following cases:

(i) $0 < R_1 < a$

[5 marks]

(ii) $R_2 > a$

[5 marks]

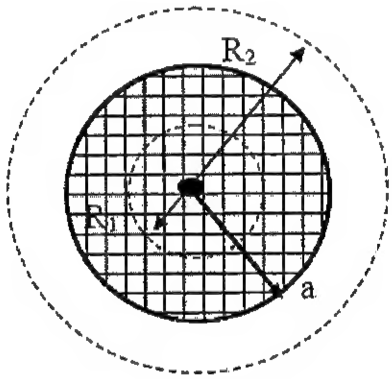

Figure 2(c)

(d) Figure 2(d) shows 2 parallel plate capacitors made of paper and plastic. Given the paper dielectric constant $\varepsilon_{r1} = 3.7$ and the plastic dielectric constant $\varepsilon_{r2} = 2$. Determine the capacitance for both configurations.
[All dimensions are in meters].

[5 marks]

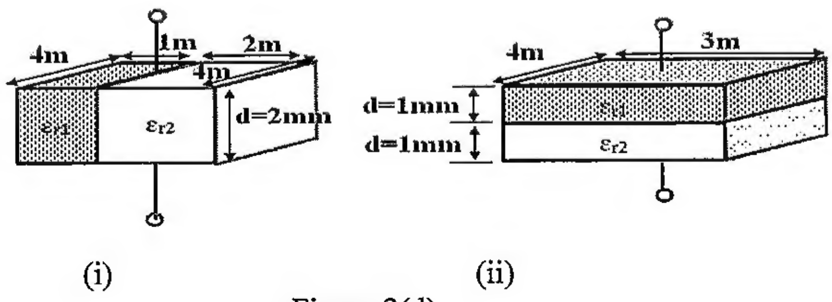

Figure 2(d)

Continued...

Question 3 (25 marks)

(a) State Ampere's Law and give its mathematical expression.

[4 marks]

(b) Figure 3(b) shows the cross section of a long coaxial cable. Current, I, flows along the positive z-direction in the inner conductor and returns through the outer conductor. The radius of the inner conductor is a, and the inner and outer radii of the outer conductor are b and c, respectively.

(i) Determine the magnetic flux density, B in each of the following THREE regions: $r \leq a$, $a \leq r \leq b$, and $b \leq r \leq c$.

[12 marks]

(ii) Plot the magnitude of magnetic flux density, B as a function of r.

[3 marks]

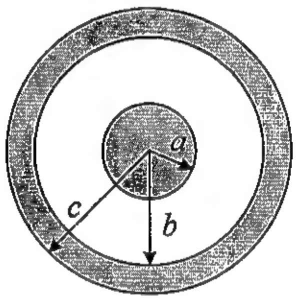

Figure 3(b)

(c) State ONE difference between electric force and magnetic force.

[2 marks]

(d) A charged particle with velocity $\vec{u}$ is moving in a medium containing uniform fields $\vec{E} = 20\hat{x}$ V/m and $\vec{B} = 3\hat{y}$ Wb/m^2. Determine $\vec{u}$ so that the particle experiences no net force

[4 marks]

Continued ...

Question 4 (25 marks)

(a) Define the following terms

(i) Permeability

[2 mark]

(ii) Reluctance

[2 mark]

(iii) Coercive force

[2 mark]

(b) A magnetic circuit is made of three different materials as shown in Figure 4(b). The relative permeability of materials 1, 2 and 3 are 2000, 1000, and 800 respectively. Material 1 has a cross section area, A_1 of (2×2) cm^2 and a length, L_1 of 10 cm. Material 2 has a cross section area, A_2 of (2×2) cm^2 and a length, L_2 of 4 cm. Material 3 has a cross section area, A_3 of (2×2) cm^2 and a length, L_3 of 6 cm. The coil carries a current, $I = 0.5$ A, has 500 turns and an air gap of 1 mm is created at material 1. By using magnetic circuit concepts (assume that fringing effect is negligible),

(i) Sketch the equivalent circuit for the magnetic circuit of Figure 4(b).

[5 marks]

(ii) Determine the total reluctance of the circuit.

[10 mark]

(iii) Calculate the magnetic flux of the circuit.

[2 mark]

(iv) If a flux density of 0.4 Tesla is required at the air gap, what should be the new supplied current?

[2 marks]

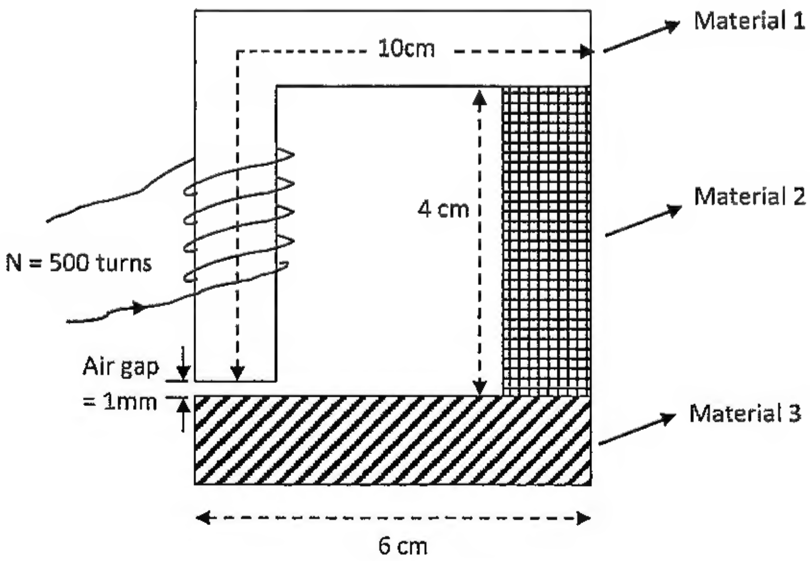

Figure 4(b)

End of Paper

Appendix A: Physical Constants, Vector and Coordinate Transformations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	1.26×10^{-6} H/m $= 4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx+\hat{y}dy+\hat{z}dz$	$\hat{r}dr+\hat{\phi}\,r\,d\phi+\hat{z}dz$	$\hat{R}dR+\hat{\theta}Rd\theta+\hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x=\hat{x}\,dydz$ $d\vec{s}_y=\hat{y}\,dxdz$ $d\vec{s}_z=\hat{z}\,dxdy$	$d\vec{s}_r=\hat{r}\,rd\phi\,dz$ $d\vec{s}_\phi=\hat{\phi}\,dr\,dz$ $d\vec{s}_z=\hat{z}\,rdr\,d\phi$	$d\vec{s}_R=\hat{R}\,R^2\sin\theta\,d\theta\,d\phi$ $d\vec{s}_\theta=\hat{\theta}\,R\sin\theta\,dR\,d\phi$ $d\vec{s}_\phi=\hat{\phi}\,R\,dR\,d\theta$
Differential volume	$(dx)(dy)(dz)$	$(dr)(r\,d\phi)(dz)$	$(dR)(R\,d\theta)(R\sin\theta\,d\phi)$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian & Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{r}=\hat{x}\cos\phi+\hat{y}\sin\phi$ $\hat{\phi}=-\hat{x}\sin\phi+\hat{y}\cos\phi$ $\hat{z}=\hat{z}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{x}=\hat{r}\cos\phi-\hat{\phi}\sin\phi$ $\hat{y}=\hat{r}\sin\phi+\hat{\phi}\cos\phi$ $\hat{z}=\hat{z}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Spherical & Cartesian	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{R}=\hat{x}\sin\theta\cos\phi$ $+\hat{y}\sin\theta\sin\phi+\hat{z}\cos\theta$ $\hat{\theta}=\hat{x}\cos\theta\cos\phi$ $+\hat{y}\cos\theta\sin\phi-\hat{z}\sin\theta$ $\hat{\phi}=-\hat{x}\sin\phi+\hat{y}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{x}=\hat{R}\sin\theta\cos\phi$ $+\hat{\theta}\cos\theta\cos\phi-\hat{\phi}\sin\phi$ $\hat{y}=\hat{R}\sin\theta\sin\phi$ $+\hat{\theta}\cos\theta\sin\phi+\hat{\phi}\cos\phi$ $\hat{z}=\hat{R}\cos\theta-\hat{\theta}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical & Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{R}=\hat{r}\sin\theta+\hat{z}\cos\theta$ $\hat{\theta}=\hat{r}\cos\theta-\hat{z}\sin\theta$ $\hat{\phi}=\hat{\phi}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{r}=\hat{R}\sin\theta+\hat{\theta}\cos\theta$ $\hat{\phi}=\hat{\phi}$ $\hat{z}=\hat{R}\cos\theta-\hat{\theta}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Appendix C: Table of Integrals

$$\int \sin^2\theta d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2 - x^2}\,dx = -\frac{1}{3}\left(a^2 - x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2 + z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2 + z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

End of Appendix